REPORT NO. RG-TR-65-7

AN ACTIVE LOW-PASS FILTER

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H. V. White

May 1965

U S ARMY MISSILE COMMAND
REDSTONE ARSENAL, ALABAMA

RSA FORM 1385, 1 JAN 63 PREVIOUS EDITION IS OBSOLETE

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DA Project No. 1X279191D678

AMC Management Structure Code No. 5282.12.127

Inertial Systems Branch
Army Inertial Guidance and Control Laboratory
Directorate of Research and Development
U. S. Army Missile Command
Redstone Arsenal, Alabama

ABSTRACT

The analysis and design of an active low-pass filter for use in inertial platform systems (though not restricted to this application) are presented in this report. The filter features minimum size and weight and increased reliability through the use of a single micro-circuit, yet it maintains performance comparable to conventional types presently used for the same purpose. Little difficulty should be encountered in making the external circuitry an integral part of the operational amplifier in a finalized version of the unit.

INTRODUCTION

The alternating current to direct current signal conversion process usually performed in an inertial platform gimbal stabilization loop requires the elimination of carrier and other undesirable frequency components from the converted signal. The filtering process should, ideally, pass all frequencies of interest with zero attenuation and phase shift and suppress all others. Additionally, modern day requirements for small size and weight and increased reliability dictate the elimination of large bulky components with numerous discrete interconnections.

A low-pass filter with characteristics approaching those described above can be realized with a single microcircuit operational amplifier and a minimum number of discrete subminiature components. Such a filter is shown in Figure 1.

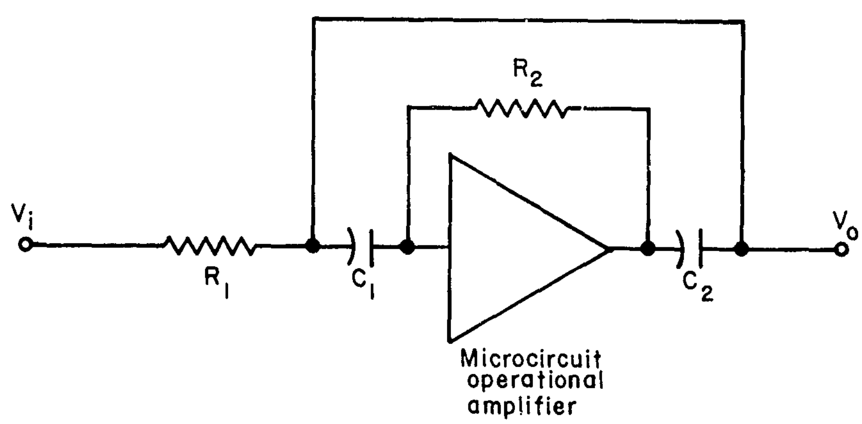

Figure 1. Active Low-Pass Filter

The filter characteristics for input frequencies of $\omega = 0$ and $\omega = \infty$ are immediately obvious. For the first case, the operational amplifier is isolated by capacitors C_1 and C_2 with the result that the input is connected directly to the output through resistor R_1. This gives $V_o = V_i$ if the filter is unloaded. For the second case, C_1 and C_2 are short circuits resulting in unity feedback from output to input such

that $V_o = 0$. The characteristics between these two extremes are best investigated by a frequency domain analysis of the filter.

ANALYSIS

The transfer function in Laplace transform notation for the system of Figure 1 is

$$\frac{V_o}{V_i} = \frac{1}{R_1R_2C_1C_2S^2 + R_1(C_1 + C_2)S^2 + 1} \tag{1}$$

The frequency response characteristics of such a second order system is well known and is shown in Figure 2 for reference.

From Figure 2, it is seen that minimum phase shift requires a small value of ζ. Corner peaking, however, increases with decreasing ζ. A compromise between these two characteristics will depend on a particular application. Figure 2 also indicates a rolloff rate of 12 decibels per octave.

If the parameters of Equation (1) are written in terms of the universal second order parameters of Figure 2,

$$R_1R_2C_1C_2 = \frac{1}{\omega_n^2} \tag{2}$$

and

$$R_1(C_1 + C_2) = \frac{2\zeta}{\omega_n} \tag{3}$$

From Equations (2) and (3), the value of C_2 is determined as

$$C_2 = \frac{\zeta}{R_1\omega_n} \pm \sqrt{\frac{\zeta^2}{R_1^2\omega_n^2} - \frac{1}{\omega_n^2 R_1R_2}} \tag{4}$$

For a real solution,

$$\frac{\zeta^2}{R_1^2\omega_n^2} \geq \frac{1}{\omega_n^2R_1R_2} \tag{5}$$

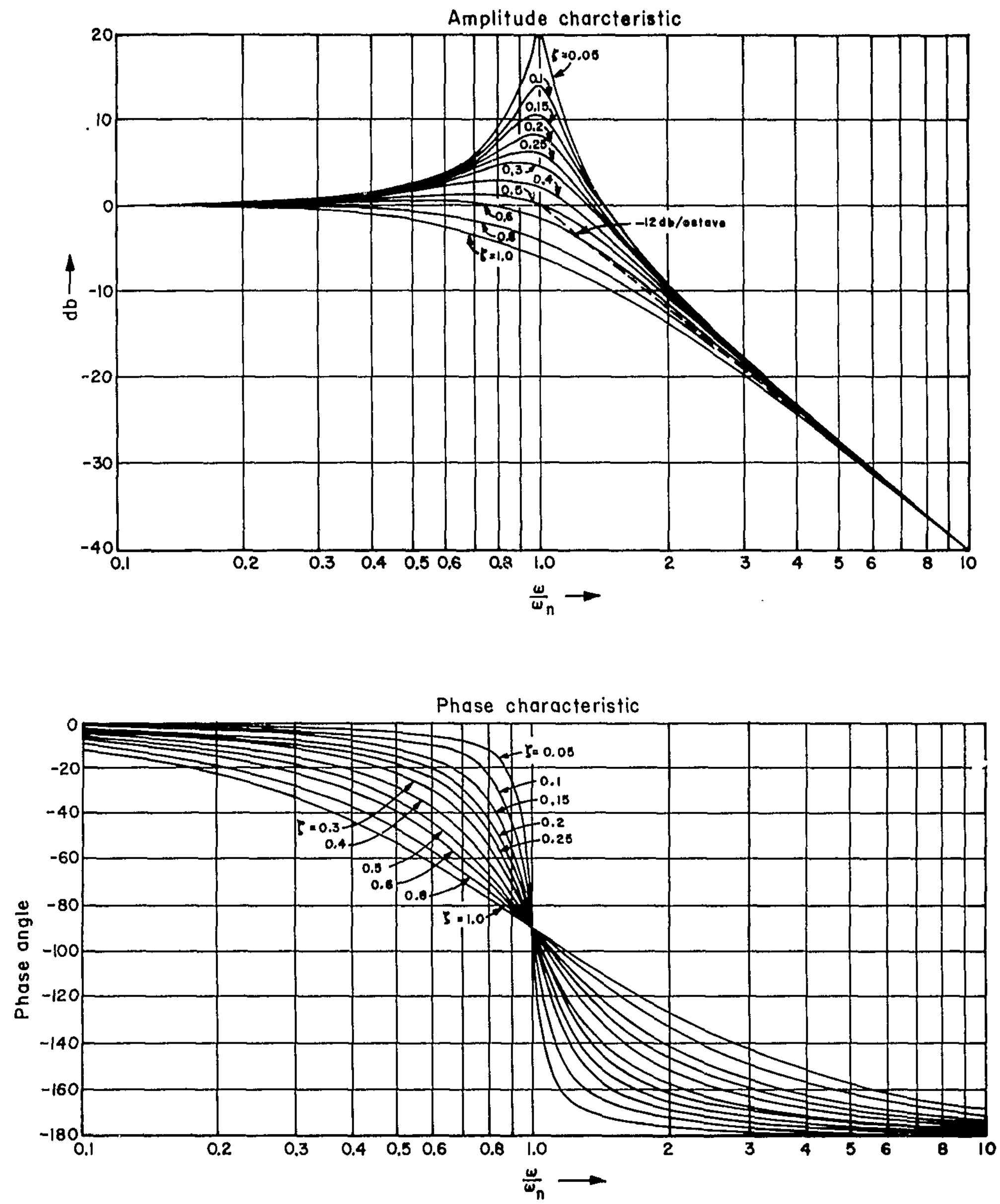

Figure 2. Normalized Frequency Response of a Second Order System

$$F(S) = \frac{1}{\frac{S^2}{\omega_n{}^2} + \frac{2\zeta}{\omega_n} S + 1} \quad \text{where } S = j\omega$$

from which

$$R_1 \leq \zeta^2 R_2 \tag{6}$$

A minimum spread in resistance values is obviously obtained when

$$R_1 = \zeta^2 R_2 \tag{7}$$

For this condition,

$$C_1 = C_2 = \frac{\zeta}{R_1 \omega_n} \tag{8}$$

with no spread in capacitance values.

Equation (8) indicates that for a given ζ and ω_n small valued capacitors may be used if R_1 is made large. Large R_1, however, is not compatible with the requirement for minimum attenuation when the filter is loaded, since the requirement on R_1 relative to R_L is

$$R_1 \ll R_L \tag{9}$$

Thus, a compromise must be made between capacitor size and the amount of allowable signal attenuation.

DESIGN EXAMPLE

The following specifications will serve to illustrate the design of a practical low-pass filter:

1) Frequencies of interest = direct current to 100 cycles per second.
2) Resonant rise $\overline{<}$ 10 decibels.
3) Phase shift at 100 cycles per second < 60 degrees.
4) Passband attenuation < −3 decibels.
5) Four hundred cycles per second attenuation $\overline{>}$ −20 decibels.
6) R_L = 15,000 ohms.

Figure 2 indicates specifications 1, 2, 3, and 5 can be approximated by choosing $\zeta = 0.2$ and $\omega_n = 2\pi(120)$ radians per second.

Under these conditions

$$C_1 = \frac{265 \times 10^{-6}}{R_1} \tag{10}$$

Since R_L = 15,000 ohms, R_1 = 1,000 ohms is chosen, thus arriving at a compromise between component size and signal attenuation, A, such that C = 0.265 microfarad and

$$A \approx \frac{R_L}{R_1 + R_L} = \frac{15,000}{16,000} = 0.94 = -0.54 \text{ decibel} \tag{11}$$

over the flat portion of the amplitude characteristic. From Equation (7), R_2 = 25,000 ohms, and the design is complete.

It is noted that more attenuation is allowable (specification 4), and therefore the value of C_1 and C_2 may be reduced if desired. Finally, it is noted that the phase shift at 100 cycles per second can be decreased by increasing the filter resonant frequency, but at the expense of decreased attenuation at 400 cycles per second.

CONCLUSIONS

The foregoing design was built and tested in this laboratory, using an operational amplifier and components specified in the Appendix. The frequency response characteristic was not significantly different from the $\zeta = 0.2$ curves of Figure 2 and is therefore not repeated here.

The filter performed as well or better than the conventional passive type presently used for the same purpose. The conventional filter is a higher order system of considerable complexity containing both capacitors and large inductors. Size and weight of the active filter are significantly reduced through the use of the microcircuits and through the the absence of the large inductors. Reliability is enhanced by a minimum number of interconnections among the external discrete components and the operational amplifier. Little difficulty should be encountered in making the external circuitry an integral part of the operational amplifier in a finalized version of the unit.

Appendix

OPERATIONAL AMPLIFIER AND COMPONENT CHARACTERISTICS

1. Operational Amplifier Characteristics

   Size — 0.675 by 0.375 by 0.1 inch flatpack.

   Supply voltage — +12 volts, −12 volts.

   Voltage gain — 73 decibels.

   Input impedance — 200,000 ohms.

   Output impedance — 200 ohms.

2. Capacitor Characteristics

Capacitors used were standard 0.27-microfarad units, ±20 percent tolerance. Subminiature versions suitable for this application are available.

3. Resistor Characteristics

Resistors used were standard $^1/_{10}$-watt units, ±10 percent tolerance. Subminiature resistors suitable for this application are available.

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1. ORIGINATING ACTIVITY *(Corporate author)* Army Inertial Guidance and Control Laboratory Directorate of Research and Development U.S. Army Missile Command Redstone Arsenal, Alabama	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

AN ACTIVE LOW-PASS FILTER

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(Last name, first name, initial)*

White, H. V.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
1 May 1965	8	

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. (DA) 1X279191D678	RG-TR-65-7
c. AMC Management Structure Code No. 5282.12.127	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
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